

# Geometrical Optics and Aberrations

**Some basics of geometrical optics and aberrations**

We will be switching now and then during the lecture to the following web site.

<http://www.micro.magnet.fsu.edu/primer/>

<http://www.micro.magnet.fsu.edu/primer/lightandcolor/lenseshome.html>

$$d_1 = v_1 \Delta t = \frac{c}{n_1} \Delta t \quad d_2 = v_2 \Delta t = \frac{c}{n_2} \Delta t$$

$$AB = \frac{d_1}{\sin I_1} = BC = \frac{d_2}{\sin I_2}$$

$$\frac{c \Delta t}{n_1 \sin I_1} = \frac{c \Delta t}{n_2 \sin I_2}$$

$$\boxed{n_1 \sin I_1 = n_2 \sin I_2} \quad \text{Snell's Law}$$

**Paraxial Approximation**

Focal Length

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{r} \quad \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

(a)

(b)

From  $\frac{\sin \theta_1}{\sin \theta_2} = \frac{s' - r}{s + r} \frac{n_2}{n_1}$  we derive:

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}$$

Combined thin lenses touching

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \dots$$

Two lenses Separation = d

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

**Single Thin Lens** → See later

Lens maker's formula

$$\frac{1}{f} = (n - 1) \left( \frac{1}{r_1} - \frac{1}{r_2} \right)$$

$n =$  index of refraction of lens  
For air  $n=1$

Positions of the principle planes

$$d_1 = f \left( \frac{1-n}{r_2} \right) \quad \text{Focal length of the thick lens}$$

$$d_2 = f \left( \frac{1-n}{r_1} \right) \quad \frac{1}{f} = (n-1) \left[ \frac{1}{r_1} + \frac{1}{r_2} - \frac{(n-1)t}{nr_1 r_2} \right]$$

The **object and image positions** are measured from the **principle planes**, H and H'

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$n =$  index of refraction of lens  
For air  $n=1$

# Aberrations

Things are not perfect;  
But they can be corrected.  
\$\$\$\$

See also

<http://www.cartage.org.lb/en/themes/sciences/physics/optics/Optical/Lens/Lens.htm>

<http://micro.magnet.fsu.edu/primer/anatomy/aberrationhome.html>

### Aberrations

General way to follow rays for Aberrations

A ray from the point  $y=h, (x=0)$  in the object passes through the optical system aperture at a point defined by its polar coordinates,  $(s, \theta)$ , and intersects the image surface at  $x', y'$ . Assume an axially symmetric object.

### Chromatic aberration

The under corrected longitudinal chromatic aberration of a simple lens is due to the blue rays undergoing a greater refraction than the red rays.

The image of an axial point in the presence of chromatic aberration is a central bright dot surrounded by a halo. The rays of light which are in focus, and those which are nearly in focus, form the bright dot, or pincushion, distortion. The out-of-focus rays form the halo. Thus, in an undercorrected visual instrument, the image would have a yellowish dot (formed by the orange, yellow, and green rays) and a purplish halo (due to the red and blue rays). If the screen on which the image is formed is moved toward the lens, the central dot will become blue; if it is moved away, the central dot will become red.

### Chromatic Aberration Correction

Lens combination (with  $n_1$  and  $n_2$ ) made of two different types of glass

$$f_1 = f \left(1 - \frac{\delta_1}{\delta_2}\right) \quad f_2 = f \left(1 - \frac{\delta_2}{\delta_1}\right)$$

$$\delta_1 = \frac{1}{n_1 - 1} \frac{dn_1}{d\lambda} \quad \delta_2 = \frac{1}{n_2 - 1} \frac{dn_2}{d\lambda}$$

Since  $dn/d\lambda$  also varies with wavelength, a lens can be achromatized over a limited wavelength interval only.

With just two lenses you can only correct over certain wavelength range.

Lateral color, or chromatic difference of magnification, results in different-sized images for different wavelengths.

### Spherical aberration

Remember, for paraxial rays  $\frac{1}{f} = (n-1) \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

A simple converging lens with under corrected spherical aberration. The rays farther from the axis are brought to a focus nearer the lens.

The image of a point formed by a lens with spherical aberration is usually a bright dot surrounded by a halo of light; the effect of spherical on an extended image is to soften the contrast of the image and to blur its details.

### Spherical Aberration

for paraxial rays  $\frac{1}{f} = (n-1) \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

$$K = \frac{f^2(n-1)}{n^2} \left[ \frac{1}{r_1^3} + \left(\frac{1}{f} + \frac{1}{r_2}\right) \left(\frac{n+1}{f} + \frac{1}{r_2}\right) \right]$$

K is a minimum when  $\frac{r_1}{r_2} = \frac{n+4-2n^2}{n+2n^2}$

In real optical instruments (cameras, microscopes) the spherical aberration is corrected with multiple lenses With different curvatures and different lens separations.

### Coma

$Coma_T = H'_{AB} - H'_P$

In the presence of coma, the rays through the outer portions of the lens focus at a different height than the rays through the center of the lens.

The coma patch. The image of a point source is spread out into a comet-shaped flare.

The relationship between the position of a ray in the lens aperture and its position in the coma patch.

### Astigmatism

The primary astigmatism of a simple lens. The tangential image is three times as far from the **Petzval surface** as the sagittal image. Note that the figure is to scale.

Astigmatism occurs when the tangential and sagittal (sometimes called radial) images do not coincide. In the presence of astigmatism, the image of a point source is not a point, but takes the form of two separate lines

**Petzval surface** = A paraboloidal surface on which point images of point objects are formed by a doublet lens whose separation is such that astigmatism is eliminated.

When the tangential image is to the left of the sagittal image (and both are to the left of the Petzval surface) the astigmatism is called negative, under corrected, or inward-(toward the lens) curving. When the order is reversed, the astigmatism is overcorrected, or backward curving.

### Distortion

Distortion. (a) Positive, or pincushion, distortion. (b) Negative, or barrel, distortion. The sides of the image are curved because the amount of distortion varies as the **cube of the distance from the axis**. Thus, in the case of a square, the corners are distorted  $2\sqrt{2}$  as much as the center of the sides.

A system which produces distortion of one sign will produce distortion of the opposite sign when object and image are interchanged. Thus a camera lens with barrel distortion will have pincushion distortion if used as a projection lens (i.e., when the film is replaced by a slide). Obviously if the same lens is used both to photograph and to project the slide, the projected image will be rectilinear (free of distortion) since the distortion in the slide will be canceled out upon projection.

### A few more things:

It goes both ways

Oblique Wave Through A Perfect Lens

Oblique Wave Through A Perfect Lens System

<http://www.microscopyu.com/tutorials/java/components/perfectlens/index.html>

<http://www.microscopyu.com/tutorials/java/components/twolensystem/index.html>

### pupils, stops, and fields

Relationships between pupils, stops, and fields.

Diaphragm #1 is the **aperture of the system which limits the size of the axial cone of energy from the object**. All of the other elements of the system are large enough to accept a bigger cone. Thus, **diaphragm #1 is the aperture stop of the system**.

The oblique ray through the center of the aperture stop is called the **principal, or chief, ray**, and is shown in the figure as a dashed line. The **entrance and exit pupils of the system are the images of the aperture stop in object and image space, respectively**.

The initial and final intersections of the dashed principal ray with the axis locate the pupils. The diameter of the axial cone of rays at the pupils indicates the pupil diameters. For any point on the object, the amount of radiation accepted by, and emitted from, the system is determined by the size and location of the pupils.

In most systems the **aperture stop** is located at or very near the objective lens. This location gives the smallest possible diameter for the objective, and since the objective is usually the most expensive component (per inch of diameter), minimizing its diameter makes good economic sense. **Field stops** could be placed at both internal images to further reduce stray radiation.

**Vignetting** in a system of separated components. The cone of rays from point *D* is limited by the lower rim of lens *A* and the upper rim of *B*, and is smaller than the cone accepted from point *C*. Note that the upper ray from *D* just passes through the image of lens *B* which is formed by lens *A*.

**Note (on axis beams):**  
 • Lens *A* is the aperture stop  
 • Lens *A* is the entrance pupil  
 • Image *C'D'* is the exit pupil

**Question: is there a well defined field stop for rays originating from *D*?**

**The apertures of an optical system as seen from point *D*.**

**$\cos^4 \theta$  "law" of illumination (energy per area)**

For off-axis image points, even when there is no vignetting, the illumination is usually lower than for the image point on the axis.

**The illumination at *H* is  $\cos^4 \theta$  times that at *A*.**

*OH* is greater than *OA* by a factor equal to  $1/\cos \theta$ , so the illumination is decreased by a factor of  $\cos^2 \theta$ . The projected area of the exit pupil at *H* is reduced by a factor which is approximately  $\cos \theta$ . Plane of illumination at *H* is oblique to the plane of recording - another reducing factor of  $\cos \theta$

The illumination at an image point is proportional to the solid angle which the exit pupil subtends from the point.

Illumination at *H* =  $(\cos^4 \theta) \times (\text{illumination at } A)$

$\cos^4 30^\circ = 0.56, \cos^4 45^\circ = 0.25, \text{ and } \cos^4 60^\circ = 0.06$

**For a lens system**  
<http://micro.sci-toys.com/NA>

Numerical Aperture  $\equiv NA = n' \sin U'$

f-Number  $\equiv \frac{1}{2NA}$

**For a fiber optic**  
[http://en.wikipedia.org/wiki/File:Optic\\_fibre-numerical\\_aperture\\_diagram.svg](http://en.wikipedia.org/wiki/File:Optic_fibre-numerical_aperture_diagram.svg)

**Numerical Aperture vs f-number**

$\theta \equiv U'$

(f-number) =  $f / D$

$NA = n \sin \theta = n \sin \left( \tan^{-1} \frac{D}{2f} \right) \approx n \frac{D}{2f}$

So (f-number)  $\approx \frac{1}{2NA}$

*n* is the index of refraction of the medium in which the lens is immersed (1.0 for air, 1.33 for pure water, and up to 1.56 for oils)

The ratio of the focal length to the clear aperture of a lens system is called the relative aperture, *f-number*, or "speed" of the system, and (other factors being equal), the illumination in an image is inversely proportional to the square of this ratio.

**f-number =  $efl/\text{clear aperture}$**

**Why is this useful?**

When a lens forms the image of an extended object, the amount of energy collected from a small area of the object is 1) directly proportional to the area of the clear aperture, or entrance pupil, of the lens. 2) At the image the illumination (power per unit area) is inversely proportional to the image area over which this object is spread. The aperture area is proportional to the square of the pupil diameter, and the image area is proportional to the square of the image distance, or square of the focal length..

Another way to look at it: **Numerical aperture =  $NA = n' \sin U'$**

*U'* is the sine of the half angle of the cone of illumination, *n'* is index of refraction  
 NA is used for microscopes; *f-#* is usually used for cameras

f-number =  $\frac{1}{2NA}$  <http://micro.magnet.fsu.edu/primer/java/imagebrightness/index.html>  
<http://micro.magnet.fsu.edu/primer/java/imageformation/airyna/index.html>

**f-number =  $\frac{1}{2NA}$**

The above relationship is only true for aplanatic systems (i.e., systems corrected for coma and spherical aberration) with infinite object distances.

The terms "fast" and "slow" are often applied to the *f-number* of an optical system to describe its "speed." A lens with a large aperture (and thus a small *f-number*) is said to be "fast," or to have a high "speed." A smaller aperture lens is described as "slow." This terminology derives from photographic usage, where a larger aperture allows a shorter (or faster) exposure time to get the same quantity of energy on the film and may allow a rapidly moving object to be photographed without blurring.

<http://micro.magnet.fsu.edu/primer/java/kohler/condenseraperture/index.html>

A system working at finite conjugates will have an object-side numerical aperture as well as an image-side numerical aperture and that the ratio  $NA/NA'$  (object-side NA)/(image-side NA) must equal the absolute value of the magnification.

### Diffraction

Diffraction of two slits or two holes

Diffraction through one hole

### Diffraction of two slits

first minimum  $\Rightarrow$

$$\sin \theta_{\min_1} = (\lambda/2)/d$$

$$d \sin \theta_{\min_n} = (\lambda/2)(n+1)$$

### Single Slit diffraction

Condition for minimum

$$a \sin \theta = m\lambda$$

$$y \approx \frac{m\lambda D}{a}$$

<http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/sinslit.html>

<http://electron9.phys.utk.edu/phys136d/modules/m9/diff.htm>

### Diffraction effects of apertures

there is a dark zone at F when

$$\sin \alpha = \frac{\pm N\lambda}{w}$$

N is an integer;  $\lambda$  is wavelength; w is the width of the slit

For a rectangular aperture, the illumination on the screen is given by

$$I = I_0 \frac{\sin^2 m_1}{m_1^2} \cdot \frac{\sin^2 m_2}{m_2^2}$$

$$m_i = \frac{\pi w_i \sin \alpha_i}{\lambda} \quad i = 1, 2$$

For a circular aperture:

$$I = I_0 \left[ \frac{2J_1(m)}{m} \right]^2$$

### Diffraction effects of apertures

INDEX  $n'$

DIFFRACTION PATTERN

$$l' = \frac{-w}{2 \sin U'}$$

When  $\alpha$  is small:  $Z = \frac{l'\alpha}{n'} = \frac{-\alpha w}{2n' \sin U'}$

**84 percent of the energy in the pattern is contained in the central spot.**  
The illumination in the central spot is almost 60 times that in the first bright ring

### Size and distribution of energy in the diffraction pattern at the focus of a perfect lens.

Ring (or band)	Circular Aperture		Slit Aperture	
	Z	Peak Illumination	Z	Peak Illumination
Central maximum	0	1.0	0	1.0
1st dark ring	$0.61 \lambda/n' \sin U'$	0.0	$0.5 \lambda/n' \sin U'$	0.0
1st bright ring	$0.82 \lambda/n' \sin U'$	0.017	$0.72 \lambda/n' \sin U'$	0.047
2d dark ring	$1.12 \lambda/n' \sin U'$	0.0	$1.0 \lambda/n' \sin U'$	0.0
2d bright ring	$1.33 \lambda/n' \sin U'$	0.0041	$1.23 \lambda/n' \sin U'$	0.017
3rd dark ring	$1.62 \lambda/n' \sin U'$	0.0	$1.5 \lambda/n' \sin U'$	0.0
3rd bright ring	$1.85 \lambda/n' \sin U'$	0.0016	$1.74 \lambda/n' \sin U'$	0.0083
4th dark ring	$2.12 \lambda/n' \sin U'$	0.0	$2.0 \lambda/n' \sin U'$	0.0
4th bright ring	$2.36 \lambda/n' \sin U'$	0.00078	$2.24 \lambda/n' \sin U'$	0.0050
5th dark ring	$2.62 \lambda/n' \sin U'$	0.0	$2.5 \lambda/n' \sin U'$	0.0

### Resolution of Optical Systems

When a bright **point source of light** is imaged, an Airy disk with a perfect lens, encircling rings appear in the image plane. The distance from the center of the Airy disk to the first dark ring is "Z":

$$Z = \frac{0.61\lambda}{n' \sin U'} = \frac{0.61\lambda}{NA} = 1.22\lambda (f/\#)$$

See <http://cnx.org/content/m13097/latest/> for some details of the derivation

The first zero occurs at  $J_1(r)/r=0$  which is at  $r=kaq/R = 3.83$  for the first ring.

$$I = I_0 \left[ \frac{2J_1(kaq/R)}{(kaq/R)} \right]^2 = I_0 \left[ \frac{2J_1(r)}{r} \right]^2 ; r=kaq/R$$

In the image the first zero  $\Rightarrow q_1 = \frac{1.22R\lambda}{2a}$  is found at

$$I(\theta) = I(0) \left[ \frac{2J_1(ka \sin \theta)}{(ka \sin \theta)} \right]^2 ; \sin \theta = q/R$$

See [http://www.matter.org.uk/tem/diffraction\\_at\\_aperture.htm](http://www.matter.org.uk/tem/diffraction_at_aperture.htm)  
This is for EM but is the same idea

### Two bright spots

**Rayleigh's criterion**, the smallest separation between two object points that will allow them to be resolved

$NA = n \sin U$

The dashed lines represent the diffraction patterns of two point images at various separations. The solid line indicates the combined diffraction pattern. Case (b) is the Sparrow criterion for resolution. Case (c) is the Rayleigh criterion.

Lord Rayleigh's criterion for resolution  $Z = \frac{0.61\lambda}{NA}$

### Diffraction limits for resolution of two sources

Light from any point in a sample that passes through a circular aperture is diffracted and this diffraction distribution in the image sets the limit of resolution on an optical device such as a microscope or telescope. The limit of resolution between two sources is when the central spot of one Airy disk is on the zero of the other Airy disk. This is known as the Rayleigh criterion. This is the most used criterion for resolution. The limits are:

$$\Delta q = 1.22R\lambda/D ; \Delta \theta = 1.22\lambda/D ; \Delta \theta = \Delta q/R$$

$D = 2a =$  diameter of the aperture ;  $R =$  distance from object to the image

<http://cnx.org/content/m13097/latest/>

**Airy Disks and Resolution**

(oil) Index of refraction  $n'$

Effect of imaging medium refractive index on diffracted orders captured by the objective. (a) Conoscopic image of objective back focal plane diffraction spectra when air is the medium between the cover slip and the objective front lens. (b) Diffraction spectra when immersion oil of refractive index similar to glass is used in the space between the cover slip and the objective front lens.

$d = 0.61 (\lambda / NA)$   
 $NA = n' \sin U'$

$NA = (n) \sin(\mu)$   
(a)  $\mu = 7^\circ$  NA = 0.12  
(b)  $\mu = 20^\circ$  NA = 0.34  
(c)  $\mu = 60^\circ$  NA = 0.87

<http://www.vanosta.be/microscopy.htm>

see <http://www.matter.org.uk/tem/resolution.htm> This is for EM but is the same idea

**Lateral resolution with and without oil**

Illustrating the difference in the angle collected with air and with oil. The larger angle will result in better lateral resolution

$d = 0.61 (\lambda / NA)$        $NA = n' \sin U'$

You need a high NA to get the best lateral resolution

<http://www.microscopyu.com/tutorials/java/imageformation/airydisk/index.html>

**Lateral resolution**

the back aperture of the objective is no longer filled at the reduced setting

Adjusting the condenser aperture directly affects spatial resolution in the microscope. Since a large aperture angle is required for maximum resolution, the front aperture of the condenser must be fully illuminated.

$NA = n' \sin U'$

**Depth of Focus**

$\Delta_{rd} = \frac{M^2 \text{R.P.}}{N.A.}$

Resolution (or strictly resolving power RP) is defined as the closest spacing of two points which can be resolved by the microscope to be separate entities.

The **depth of focus** is the amount by which the image may be shifted longitudinally with respect to some reference plane (e.g., film, reticle) and which will introduce no more than the acceptable blur.

The **depth of field** is the amount by which the object may be shifted before the acceptable blur is produced.

Linear angular blur:  $\beta = \frac{B}{D} = \frac{B'}{D'}$  (for the object and image side)

**Axial Intensity Distribution**

The **depth of field** depends on the resolution of the individual microscope.

The **depth of focus** depends on the magnification.

The depth of focus can be improved by increasing your magnification. Both resolution and magnification depend on the NA. By using a high-quality lens and increasing magnification, the focus depth is greatly improved.

Figure

<http://micro.magnet.fsu.edu/primer/anatomy/focusdepth.html>

### depth of field for a system with a clear aperture $A$

Table 9 Depth of Field and Image Depth \*

Magnification	Numerical Aperture	Depth of Field ( $\mu$ )	Image Depth (mm)
4x	0.10	55.5	0.13
10x	0.25	8.5	0.80
20x	0.40	5.8	3.8
40x	0.65	1.0	12.8
60x	0.85	0.40	29.8
100x	0.95	0.19	80.0

\* Source: Nikon

For diffraction-limited optics, the wave-optical value of  $Z$  is

$$Z = \sqrt{n^2 - NA^2} \lambda / NA^2 \approx n\lambda / NA^2$$

The **depth of field** is determined by the **distance** from the **nearest object plane** in focus to that of the **farthest plane** also simultaneously in focus.

The pinhole camera has an infinitely small NA and an infinite depth of field— all objects, both near and far, are simultaneously in focus in such a camera.

<http://www.microscopyu.com/articles/formulas/formulasfielddepth.html>

### Putting them together – rough guide

Figure 2

$\pm D.O.F. = \frac{\omega \times 250,000}{N.A. \times M} + \frac{\lambda}{2(N.A.)^2}$  ( $\mu\text{m}$ )

D.O.F.: Depth Of Focus  
 $\omega$ : Resolving power of eye 0.0014 (when optical angle is 0.6 degrees)  
 $M$ : Total magnification (objective lens magnification  $\times$  eyepiece magnification)  
 $\lambda$ : Wavelength (550nm)

$\rightarrow \pm D.O.F. = \frac{350}{N.A. \times M} + \frac{0.275}{(N.A.)^2}$

This indicates that the focal depth becomes smaller as the numerical aperture becomes larger!

Example With MPFL1100X (N.A.=0.90), WFN110X:  
 $\pm D.O.F. = \frac{350}{0.90 \times 1000} + \frac{0.275}{0.91} = 0.38 + 0.34 = 0.72\mu\text{m}$

[http://www.olympus-ims.com/en/microscope/terms/focal\\_depth/](http://www.olympus-ims.com/en/microscope/terms/focal_depth/)

For a video camera, the focal depth will vary according to number of pixels of CCD, optical magnification, and numerical aperture.

### Gaussian laser beam focusing

$I(r) = I_0 e^{-2r^2/w^2}$  <http://www.cvimellesgriot.com/products/Documents/TechnicalGuide/Gaussian-Beam-Optics.pdf>

$I(r)$  = the beam intensity at a distance  $r$  from the beam axis  
 $I_0$  = the intensity on axis  
 $r$  = the radial distance  
 $e = 2.718...$   
 $w$  = the radial distance at which the intensity falls to  $I_0/e^2$ , i.e., to 13.5 percent of its central value. This is usually referred to as the beam width, although it is a semi-diameter. It encompasses 86.5 percent of the beam power.

Total power

$$P_{\text{tot}} = \frac{1}{2} \pi I_0 w^2$$

Power through a circular aperture

$$P(a) = P_{\text{tot}} (1 - e^{-2a^2/w^2})$$

The spreading of a Gaussian Beam (most lasers are Gaussian)

As the laser beam (wavelength  $\lambda$ ) spreads it progresses along  $z$  as:

$$w_z^2 = w_0^2 \left[ 1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2 \right]$$

The angular spread of the beam is:

$$\frac{\alpha}{2} = \frac{w_z}{z} \approx \frac{\lambda}{\pi w_0}$$

$$\alpha = \frac{4\lambda}{2\pi w_0} = \frac{1.27\lambda}{\text{diameter of the beam}}$$

Note this is almost the same as the diffraction of a parallel beam of equal intensity passing through a hole of diameter  $D$ :

$$\alpha_c = \frac{1.22\lambda}{D}$$

So, now let's look at a microscope

<http://www.microscopyu.com/articles/formulas/index.html>

### Useful links we will use throughout the lecture

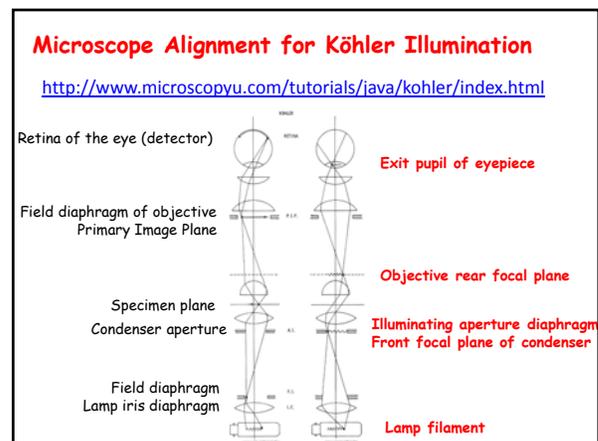
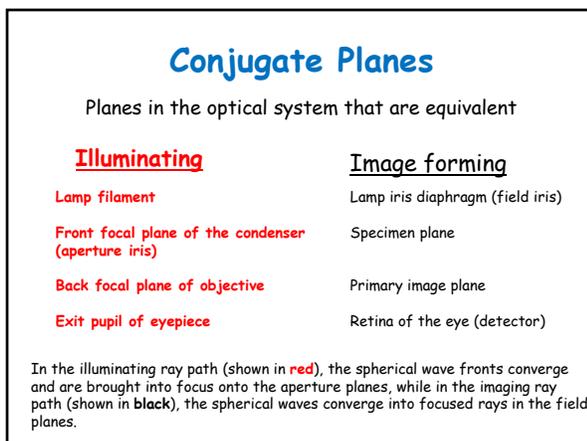
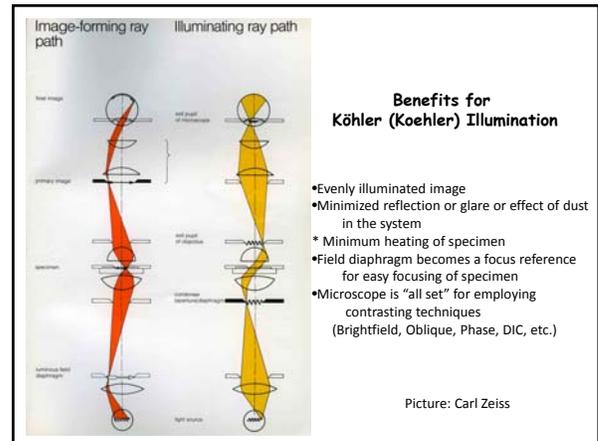
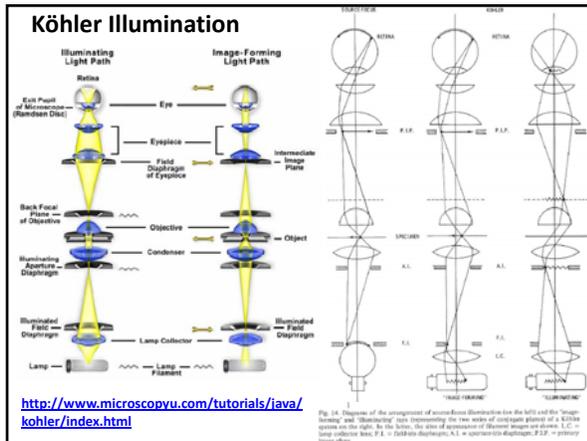
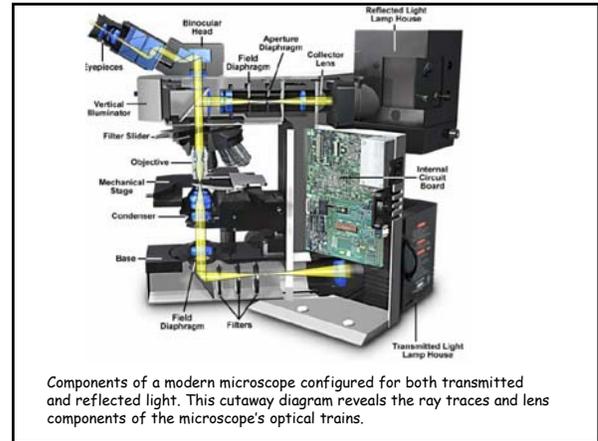
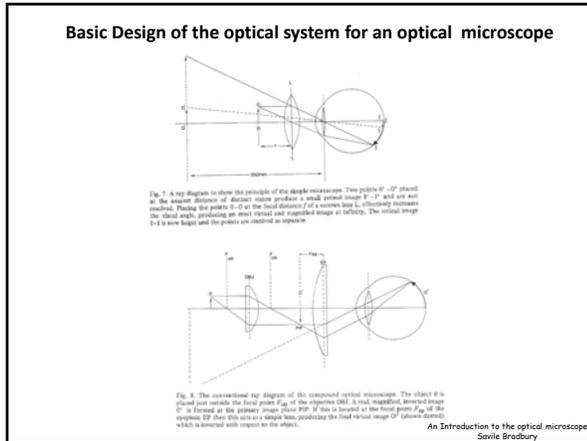
<http://www.microscopyu.com/tutorials/java/aberrations/curvatureoffield/index.html>

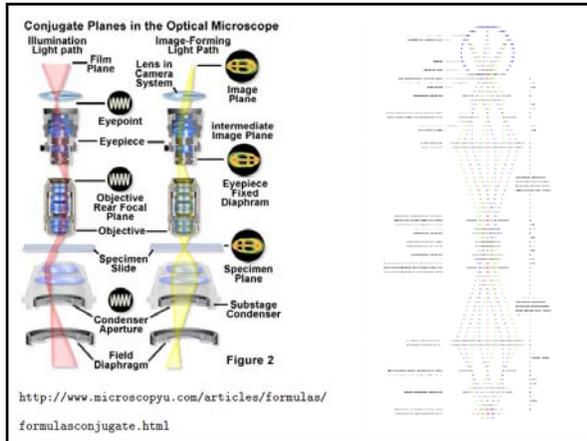
<http://www.microscopyu.com/articles/formulas/formulasconjugate.html>

<http://www.microscopyu.com/articles/dic/index.html>

<http://micro.magnet.fsu.edu/primer/techniques/dic/dicconfiguration.html>

<http://www.olympusmicro.com/primer/techniques/dic/dichome.html>



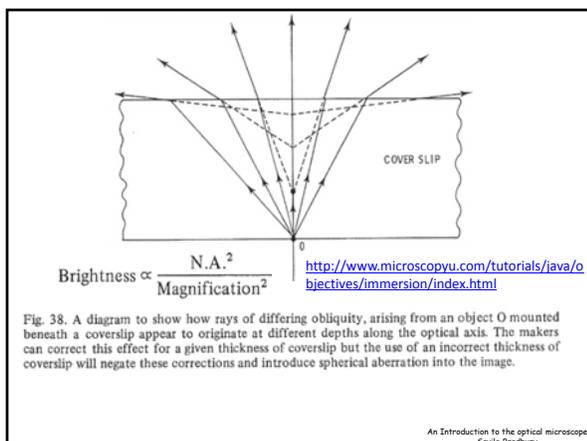
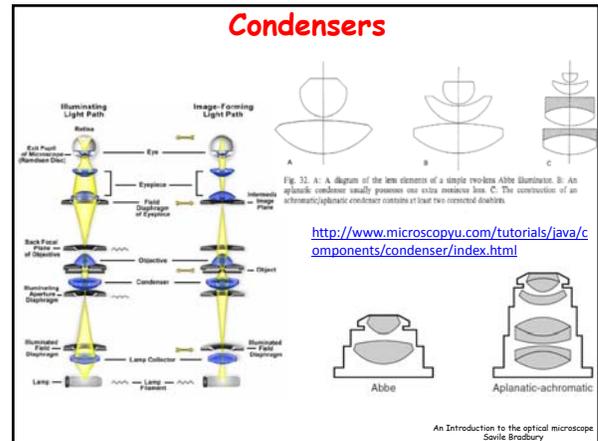
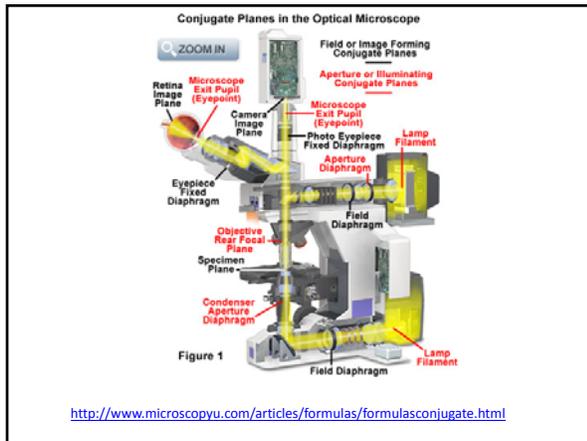


### Class questions

- 1) Where can you place a reticule?
- 2) Where do you place the phase plates?

answer after discussing a phase plate

<http://www.microscopyu.com/tutorials/java/conjugateplanes/index.html>



- STEP 1** Focus your sample in brightfield. Note the dark shadow in the upper right.
- STEP 2** Close the field diaphragm so it looks something like this:
- STEP 3** Focus the edge of the diaphragm by adjusting the condenser height, so it looks like this:  
If the image moves out of your field of view, skip to step 4, then come back to step 3.)
- STEP 4** Center the image using the two centering screws, so it looks like this:  
(Note centered, crisp edge)
- STEP 5** Open the field diaphragm until it is at the edge of the field of view. (Note that the shadow in step 1 is gone.)

[http://www.microscopy.olympus.eu/microscopy/es/39\\_9216.htm](http://www.microscopy.olympus.eu/microscopy/es/39_9216.htm)

Aligning the condenser lens to assure Koehler illumination is optimal. This is important for transmitted light work: bright field, phase or DIC.

See also:  
[http://www.imagingandanalysis.com/Koehler\\_illumination.pdf](http://www.imagingandanalysis.com/Koehler_illumination.pdf)

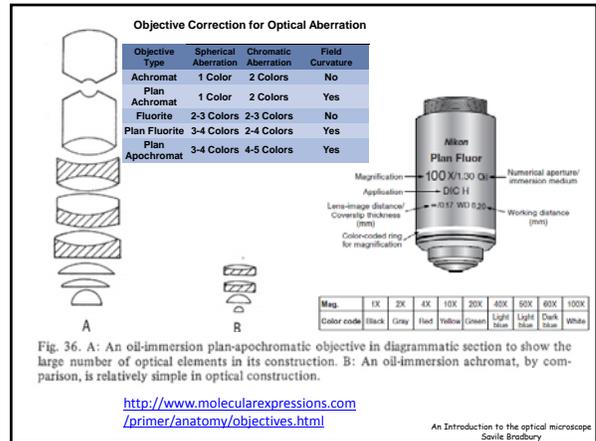
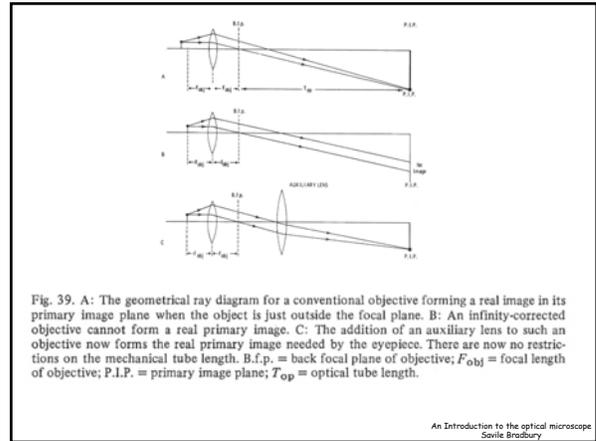
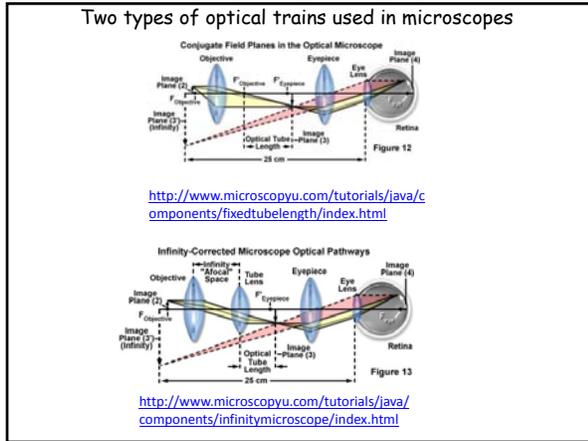
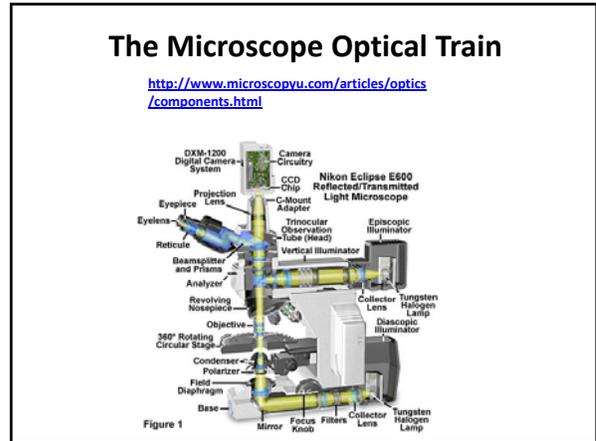
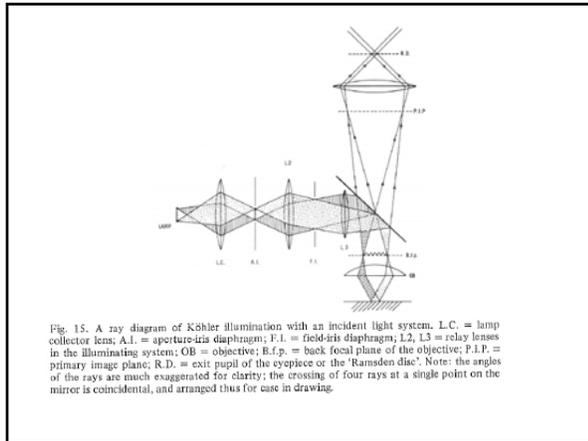
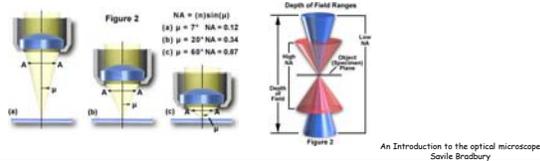


Table 1. Some typical data for objectives

Objective type	Initial mag.	Focal length (mm)	N.A.	Working distance (mm)	Approx. resoln. (μm)
Plano	X 1	33	0.04	30	7
Plano	X 2.5	56	0.08	11	3.4
Achromat	X 4	32	0.12	24	2.5
Planachromat	X 10	16	0.25	0.6	1.1
Planachromat	X 40	4	0.65	0.15	0.42
Planapo	X 40	4	0.75	0.44	0.37
Apochromat	X 40	4.6	0.95	0.09	0.30
Achromat*	X 63	2.9	0.85	0.24	0.32
Planachromat*	X 100	1.7	1.30	0.26	0.21
Planapo*	X 100	2.4	1.32	0.24	0.20

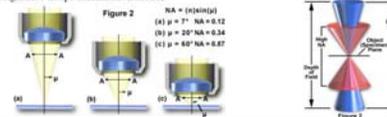
\*Signifies oil-immersion objective.



Characteristics of Selected Objective Lenses\*

M	Type	Medium (n)	WD (mm)	NA	d <sub>min</sub> (μm)	DOF (μm)	B
5	Achromat	1	9.9	0.12	2.80	38.19	0.1
10	Achromat	1	4.4	0.25	1.34	8.80	0.4
20	Achromat	1	0.53	0.45	0.75	2.72	1.0
25	Fluorite	1.515	0.21	0.8	0.42	1.30	6.6
40	Fluorite	1	0.5	0.75	0.45	0.98	2.0
40	Fluorite	1.515	0.2	1.3	0.26	0.49	17.9
60	Apochromat	1	0.15	0.95	0.35	0.61	2.3
60	Apochromat	1.515	0.09	1.4	0.24	0.43	10.7
100	Apochromat	1.515	0.09	1.4	0.24	0.43	3.8

\*The magnification (M), type of lens design, refractive index (n) of the intervening medium (air or immersion oil), working distance (WD), numerical aperture (NA), minimum resolvable distance (d), depth of field (DOF), and brightness (B) are indicated. Terms are calculated as: wave-optical depth of field,  $n/NA^2$ ; brightness in epi-illumination mode.



### Microscope Slides and Coverslips

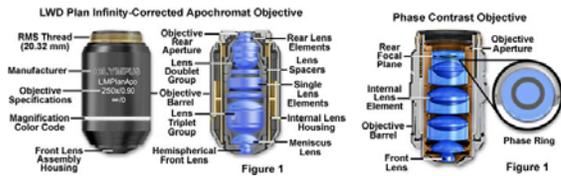
Many objectives are designed to be used with standard (1.1 mm thick) glass slides and coverslips of a certain thickness, usually 0.17 mm, which corresponds to thickness grade 1.5. Other coverslip thicknesses induce spherical aberration and give poorer performance, especially when used with high, dry lenses above 40. For lenses with an NA 0.4, coverslip thickness is not particularly important.

Grade Number	Thickness (mm)
0	0.083–0.13
1	0.13–0.16
1.5	0.16–0.19 (standard)
2	0.19–0.25

### Information on Objectives

<http://www.molecularexpressions.com/search/index.asp>

"click on the URL, and then enter "objectives"



### Different microscope imaging modes

Type	Feature	Appearance	Uses
Bright-field	Visible light	Colored/clear specimen - light background	Stained specimens/live ones with sufficient color contrast
Dark-field	Visible light	Bright specimen on dark background	Unstained or difficult to stain specimen
Phase-Contrast	Visible light / Phase shifting plate in objective with special condenser	Different degrees of brightness and darkness	Observe internal structure of specimen
Differential Interference contrast	Visible light out of phase	Nearly 3-D image	Observe finer detail of internal structure of unstained specimen
Fluorescence	UV-light excites molecules	Bright, fluorescent specimen	Detection of organisms or antibodies in clinical specimens
Transmission Electron microscope	Electron beam/magnetic lens	Highly magnified	Internal structures; viruses
Scanning Electron microscope	Electron beam	Three-dimensional view of surfaces	External or internal surfaces of cells
Scanning Tunneling	Wire probe over surfaces	Three-dimensional view of surfaces	Observation of external surfaces of atoms or molecules

## Bright field And Dark field Illumination

<http://www.molecularexpressions.com/primer/anatomy/illuminationhome.html>

Stauroneis phoenicenteron (720 nm frustule spacing)

<http://www.microscopy-uk.org.uk/mag/artmar06/go-phase.html>

### Bright Field Image - no staining

An amoeba with the numerous crystals that are found in this species.  
We will use this example throughout this lecture to compare methods

<http://www.microscopy-uk.org.uk/index.html>?http://www.microscopy-uk.org.uk/intro/index.html

Untinted and Stained Specimens in Brightfield Illumination

Figure 2

Figure 2

Köhler Illumination for Brightfield Reflected Light

### Dark field illumination

solid lines = illumination  
dashed lines = scattering or fluorescence

### Dark Field illumination

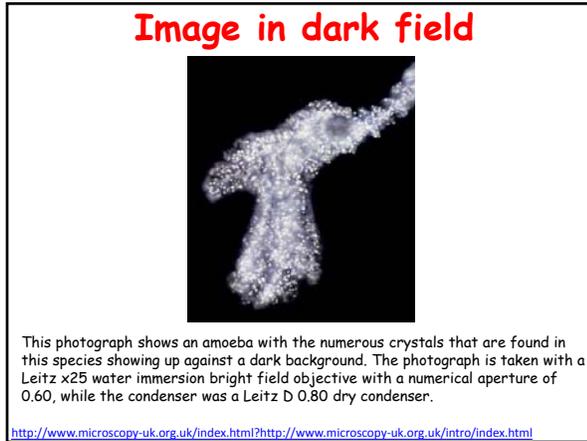
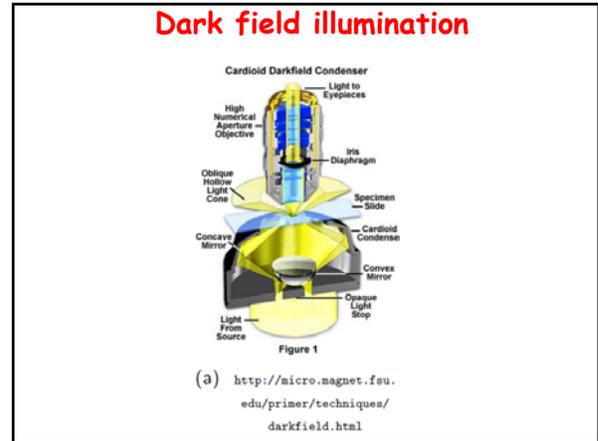
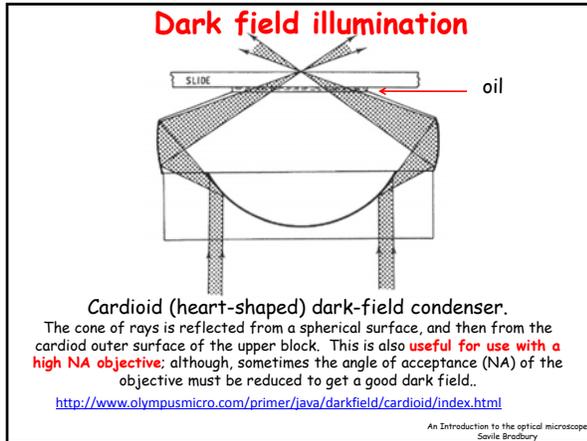
The illumination light is occluded from the central optical axis of the condenser, and this light is focused onto the sample, and this light that is not diffracted passes outside of the acceptance angle of the objective. The objective gathers light (and an image) of the diffracted, scattered or fluorescence light that is within the acceptance angle of the objective. The image is in "reverse contrast". **This works well for lower NA objectives, but is more difficult for higher NAA objectives.**

An Introduction to the optical microscope  
Saville Bradbury

### Dark field illumination

A paraboloid dark-field condenser. The condenser is a solid piece of glass, in the shape of a parabola (flat and polished on top). Direct parallel light is occluded from the center. The glass sides are totally reflect the light, and focus at the sample. In order to focus correctly, oil must be used between the slide and the top of the condenser. **This condenser is useful for very high power objectives.**

An Introduction to the optical microscope  
Saville Bradbury



## First a look at Abbe's theory of image formation Fourier Optics

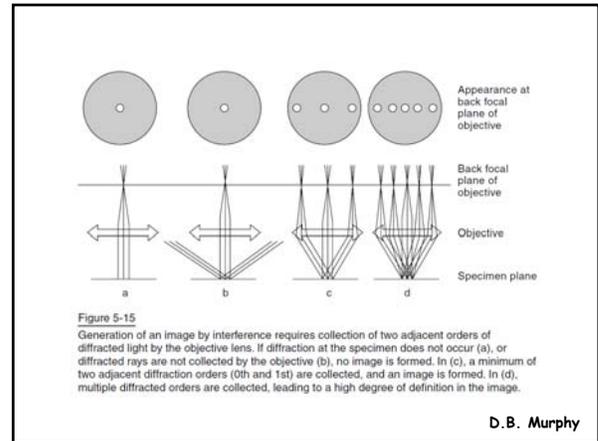
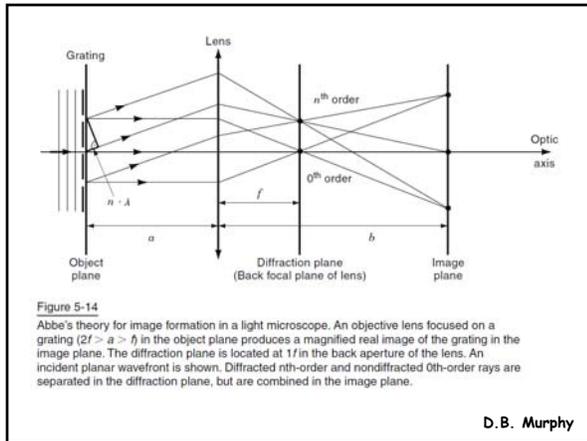
Nice elementary description in:  
**Fundamentals of Light Microscopy  
and Electronic Imaging**  
D. B. Murphy

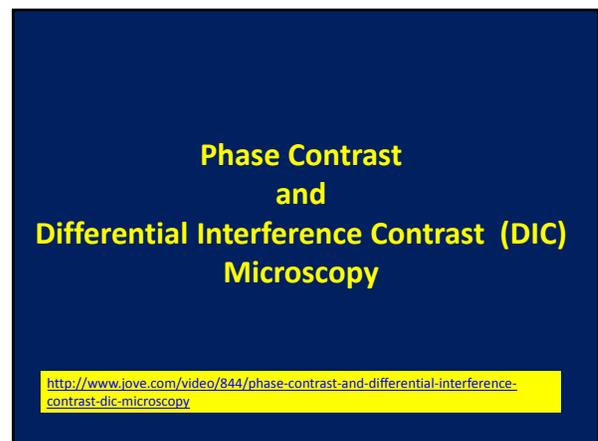
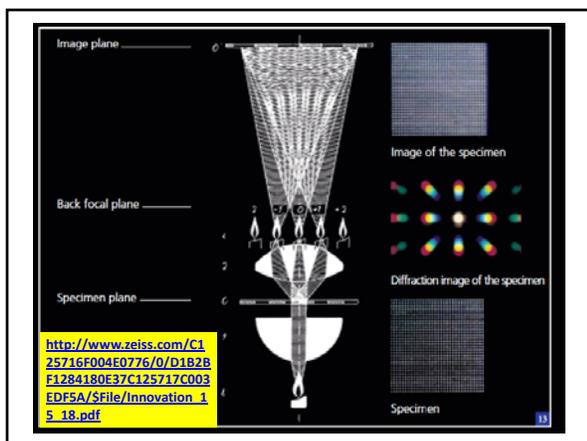
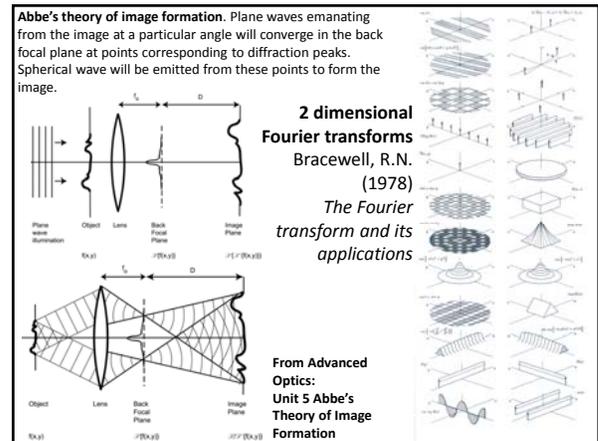
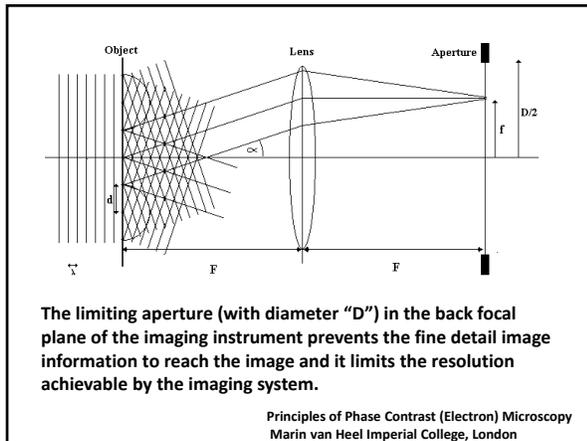
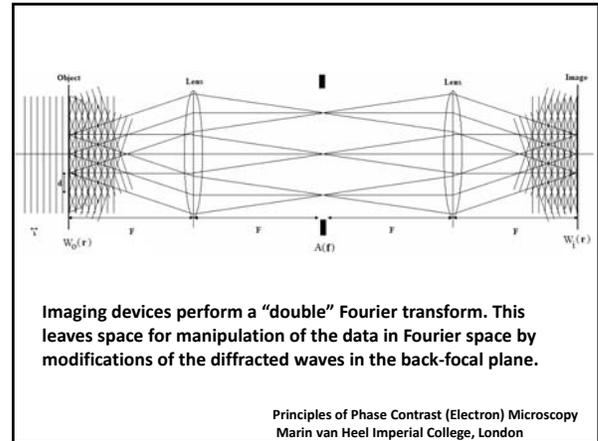
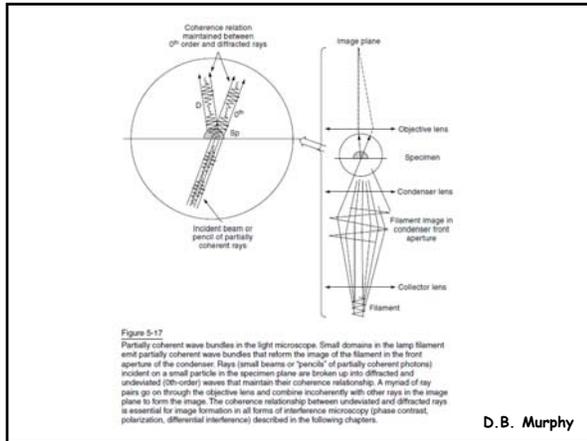
Or see more advanced in:  
**Optical Physics**  
S. G. Lipson, H. Lipson and D.S. Tannhauser

**See also:**

An Elementary Theoretical Approach to the Abbe Theory of  
Optical Image Formation

ANTHONY GERRARD  
*Bristol College of Science and Technology, Ashley Down, Bristol, England*  
(Received 19 December 1962)

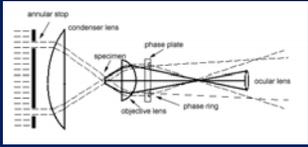




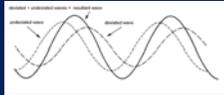
## Phase Contrast



"The microscope image is the interference effect of a diffraction phenomenon." Abbe



The Nobel Prize in Physics 1953 was awarded to Frits Zernike "for his demonstration of the phase contrast method, especially for his invention of the phase contrast microscope".



PHASE CONTRAST,  
A NEW METHOD FOR THE MICROSCOPIC  
OBSERVATION OF TRANSPARENT OBJECTS  
PART II \*)  
by F. ZERNIKE, Groningen  
Physica IX, no 10 December 1942

Fig. 19–20. Salivary chromosomes of *Drosophila*, 0.65 objective, enl. 2200 × ; fig. 19 bright ground with narrow illuminating cone, fig. 20 positive phase contrast.

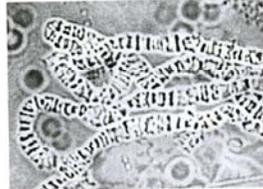



Fig. 19 Fig. 20

### How I Discovered Phase Contrast

F. Zernike  
Department of Physics, University of Groningen, Netherlands  
Science (1955) 121, 345

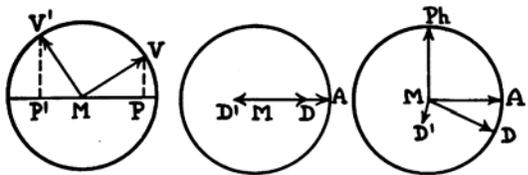


Fig. 1. Vector diagrams showing the interplay of phases.

PHASE CONTRAST,  
A NEW METHOD FOR THE MICROSCOPIC  
OBSERVATION OF TRANSPARENT OBJECTS  
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by F. ZERNIKE, Groningen  
Physica IX, no 10 December 1942

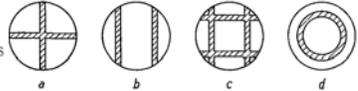
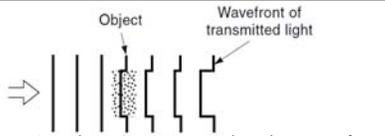


Fig. 9. Various forms of phase strips suggested.

### How I Discovered Phase Contrast

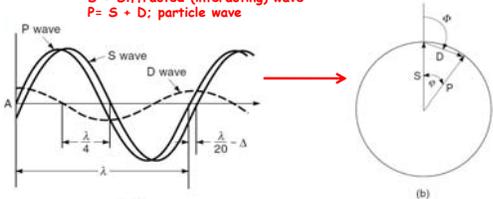
F. Zernike  
Department of Physics, University of Groningen, Netherlands  
Science (1955) 121, 345

On looking back to this event, I am impressed by the great limitations of the human mind. How quick we are to learn—that is, to imitate what others have done or thought before—and how slow to understand—that is, to see the deeper connections. Slowest of all, however, are we in inventing new connections or even in applying old ideas in a new field.



Disturbance by a phase object to an incident planar wave front.

S = undeviated (non-interacting) source (surround) wave  
D = Diffracted (interacting) wave  
P = S + D; particle wave



Phase relations between S, D, and P waves in bright-field microscopy, P=S+D

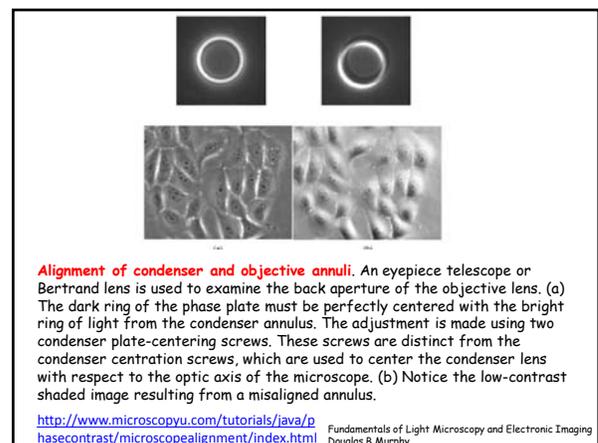
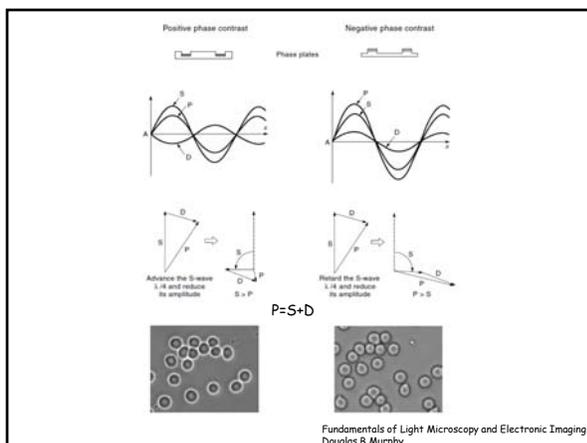
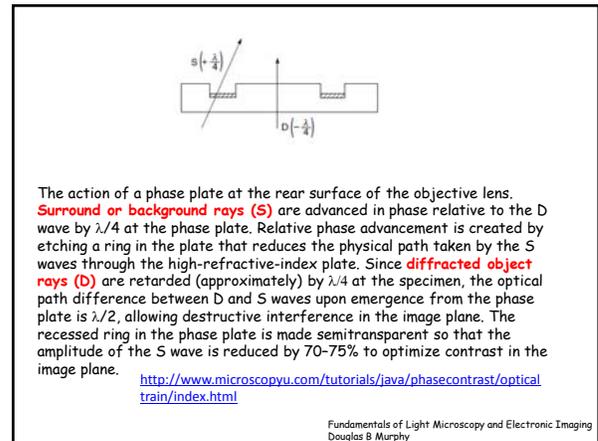
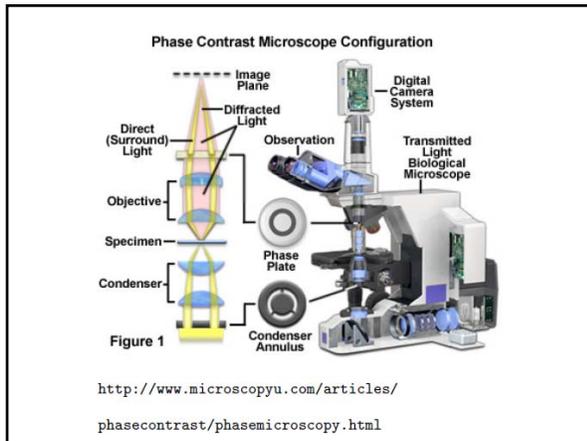
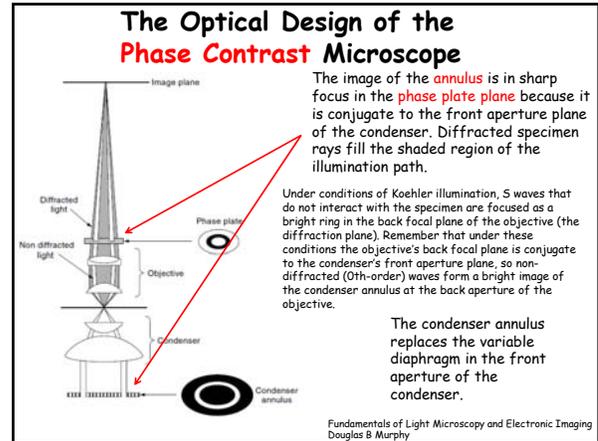
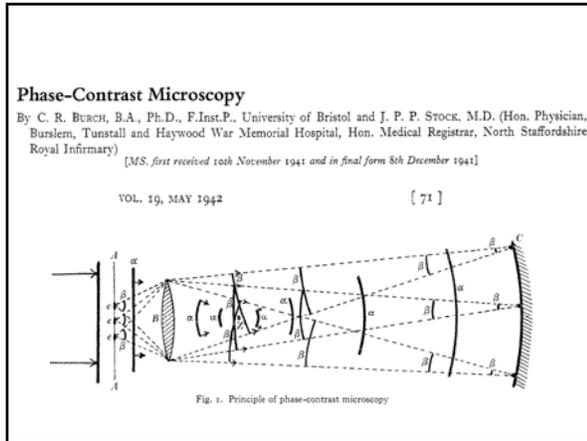
Fundamentals of Light Microscopy and Electronic Imaging  
Douglas S. Murphy

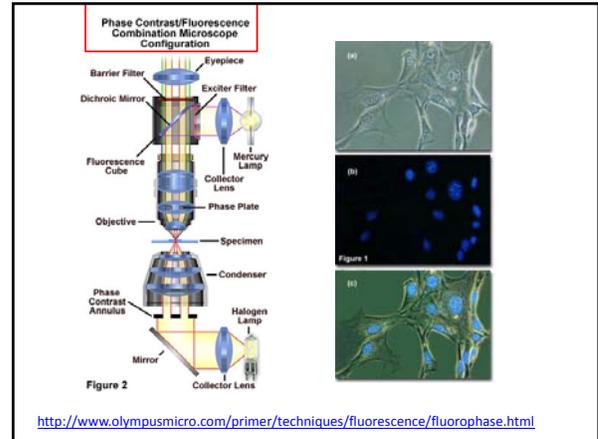
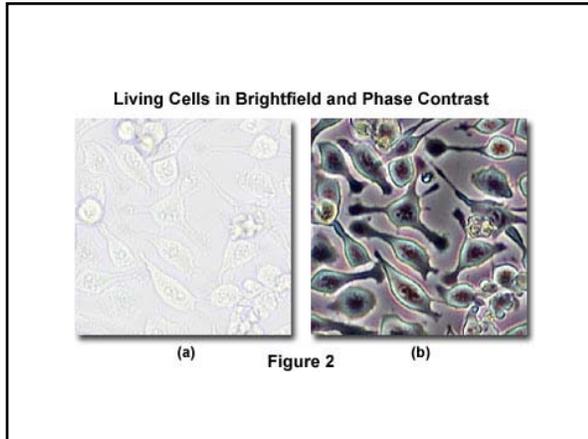
## Double refraction or birefringence

optical path difference  $\Delta = (n_2 - n_1)t$   
 $t =$  thickness of the object

Relative retardation and birefringence are related by  $\Gamma = (n_E - n_O)t$

optical phase difference  $= \delta = 2\pi \frac{t}{\lambda_0} (n_2 - n_1) = 2\pi \frac{t}{\lambda_0} (n_E - n_O) = 2\pi \frac{\Delta}{\lambda_0}$





<http://www.microscopy-uk.org.uk/index.html?http://www.microscopy-uk.org.uk/intro/index.html>

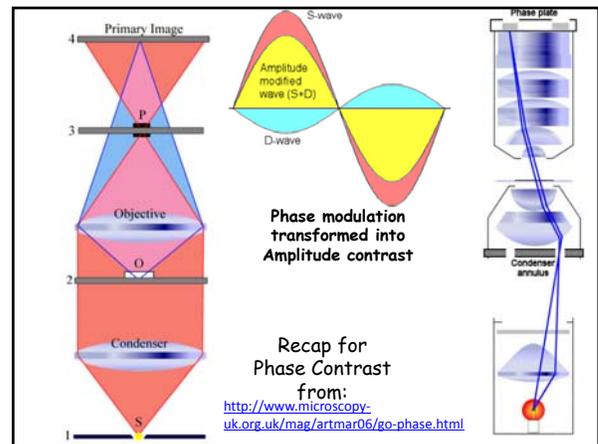
An amoeba with the numerous crystals that are found in this species.

<http://www.microscopy.com/articles/phasecontrast/index.html>

<http://www.olympusmicro.com/primer/techniques/phasecontrast/phaseindex.html>

There are some limitations of phase contrast microscopy:

- Phase images are usually surrounded by halos around the outlines of details. Such halos are optical artifacts, which sometimes obscure the boundaries of details.
- The phase annuli do limit the working numerical aperture of the optical system to a certain degree, thus reducing resolution.
- Phase contrast does not work well with thick specimens because of shifts in phase occur from areas slightly below or slightly above the plane that is in focus. Such phase shifts confuse the image and distort image detail.
- Phase images appear gray if white light is used and green if a green filter is used. In the past, many microscopists restricted their film to black and white when performing photomicrography on phase specimens. Today, many color films reproduce black, white, and grayscales very effectively.



Unlike *differential interference contrast* (see next slides) and **Hoffman modulation contrast**, the circular geometry of **phase contrast** illumination and detection enables specimen observation **without orientation-dependent artifacts**. Phase contrast is also insensitive to polarization and birefringence effects, which is a major advantage when examining living cells growing in plastic tissue culture vessels.

<http://www.olympusmicro.com/primer/techniques/hoffman.html>

# DIC

## Differential interference contrast (Nomarski) imaging is a modification of phase microscopy.

Nomarski imaging is a phase imaging technique that gives good rejection of out-of-focus interference. It provides an image of the gradient (i.e. spatial rate of change) of refractive index inhomogeneities in the sample.



Georges Nomarski

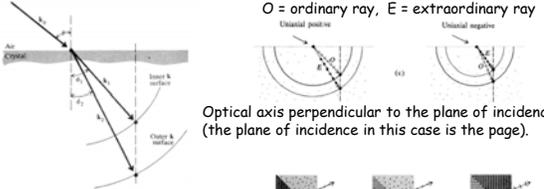
<http://www.microscopyu.com/articles/dic/index.html>  
<http://microscopy.berkeley.edu/Resources/instruction/DIC.html>

# Light propagation in crystals

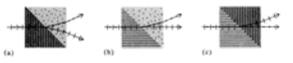
## Propagation of light in crystals

When light propagates down the optical axis, the index of refraction does not depend on the polarization direction

O = ordinary ray, E = extraordinary ray



Optical axis perpendicular to the plane of incidence (the plane of incidence in this case is the page).

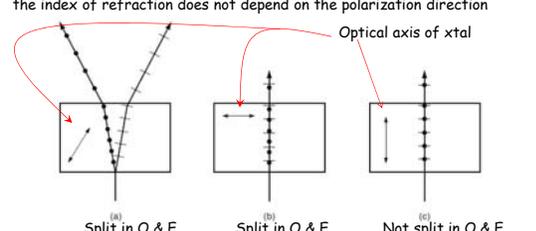


Wave vectors for **double refraction** at the boundary of a crystal

Three ways to separate unpolarized light into two divergent rays of orthogonal polarization. b = Wollaston prism.

### Splitting of an incident ray into O- and E-ray components by a birefringent crystal.

When light propagates down the optical axis, the index of refraction does not depend on the polarization direction



Optical axis of xtal

(a) Split in O & E      (b) Split in O & E      (c) Not split in O & E

(a) Optic axis at oblique angle to incoming light ray  
 (b) An incident ray whose propagation axis is **perpendicular** to the optic axis  
 (c) An incident ray whose propagation axis is **parallel** to the optic axis is not split

Different indices of refraction and therefore different speeds,  $n_O \sin \phi_O = \sin \theta$   
 $n_E \sin \phi_E = \sin \theta$

## Double refraction or birefringence

optical path difference  $\Delta = (n_2 - n_1)t$   
 $t =$  thickness of the object

Relative retardation and birefringence are related by  $\Gamma = (n_E - n_O)t$

optical phase difference  $= \delta = 2\pi \frac{t}{\lambda_0} (n_2 - n_1) = 2\pi \frac{t}{\lambda_0} (n_E - n_O) = 2\pi \frac{\Delta}{\lambda_0}$

### E & O wave travelling through a birefringent object

Waves resulting from the combination of superimposed O and E rays have **elliptical, spherical, or planar waveforms**, depending on the amount of relative phase shift between the two rays.

$$\text{optical phase difference} = 2\pi \frac{t}{\lambda_0} (n_E - n_O)$$

Phase retardation ( $\Gamma$ ) and resultant wave form leaving object

Amplitude of transmitted component at analyzer

<http://www.molecularexpressions.com/primer/techniques/polarized/compensatorshome.html>

de Sénarmont Compensator Configuration

Figure 2

<http://www.molecularexpressions.com/primer/techniques/polarized/quarterwavelengthplate.html>

<http://www.molecularexpressions.com/primer/techniques/polarized/desenarmontcompensator.html>

The principle behind the de Sénarmont compensation technique rests on the fact that the elliptically (or circularly) polarized light electric vector emerging from the specimen is superimposed upon the circular polarization vector introduced by the quarter wavelength plate to yield plane (linearly) polarized light having a vibration azimuth different from that of the polarizer (see Figure 2). The effect occurs because the quarter wavelength plate produces linearly polarized light from elliptically or circular polarized light that is incident on the crystal surface. The azimuth of the linear polarized light emerging from the compensator is a direct function of the optical path difference induced by the specimen. By rotating the analyzer until the specimen is extinct (dark), the azimuth of the vibration produced by the de Sénarmont compensator can be ascertained along with the optical path difference introduced by the specimen.

Optic Axis

Optical Cement

Unpolarized Input

o-Polarized e-Ray

e-Polarized o-Ray

### Wollaston polarizing prism

It usually consists of two calcite right-angle prisms optically cemented together at the hypotenuse. The optic axis of each section is orthogonal to that of the other section. **An unpolarized ray traversing the first prism section is not split, but the o-ray is retarded with respect to the e-ray.** The o-ray vibrates parallel to the optic axis and the e-ray perpendicular to the optic axis. Upon entering the second section, the o-ray becomes the e-ray, and vice versa. The o-ray, now slower, is bent toward the interface normal, and the e-ray is bent away from the interface normal. Prisms with a deviation angle from about 5 to 45 deg between the exit beams can be obtained, depending on the right-angle prism base angles. For very high-power applications, the prisms may not be cemented together, resulting in a reduction of transmission.

Optic Axis

Interference Plane

e-Ray

o-Ray

Optic Axis

### Nomarski polarizing prism

The optic axis of the first right-angle calcite prism is skewed as shown, while the optic axis of the second prism is oriented the same as for the Wollaston prism. This angled optic axis causes the ordinary and extraordinary rays to intersect outside the prism, forming an interference plane. The resulting phase shifts can be detected by an analyzer. The exact distance of this interference plane from the prism is determined by the angle of the skewed optic axis and is set by the manufacturer. Normarski prisms are used in differential interference contrast (DIC) microscopes.

## Implementation in DIC microscopy

OPD (nm)

Perceived intensity

In phase

Out of phase

Gradients in optical path length yield differences in amplitude.

<http://www.molecularexpressions.com/primer/techniques/dic/dichome.html>

Fundamentals of Light Microscopy and Electronic Imaging  
Douglas B. Murphy

**Math digression**

$f(x, y)$  is the E&M amplitude, and  $I(x, y)$  is the intensity  
We are measuring incoherent light (the intensity)

$$I(x, y) = |f(x, y) - f(x + \delta x)|^2 \otimes p(x, y) ; p(x, y) = \text{point spread function}$$

$$f(x + \delta x, y) \approx f(x, y) + \delta x \frac{\partial f}{\partial x}$$

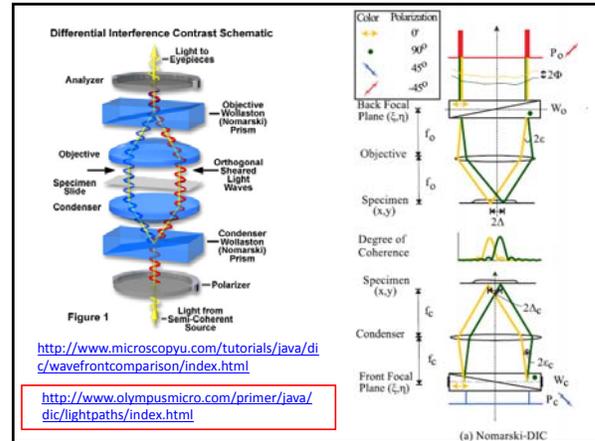
$$I(x, y) \approx \delta x^2 \left| \frac{\partial f}{\partial x} \right|^2 \otimes p(x, y)$$

$$f = |f| \exp(i\phi(x, y))$$

$$\frac{\partial f}{\partial x} = i f \frac{\partial \phi}{\partial x} + \exp(i\phi(x, y)) \frac{\partial |f|}{\partial x}$$

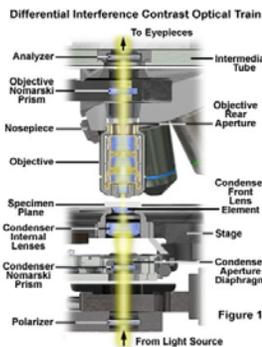
If the object is dominated by phase changes, and not amplitude changes, then:

$$I(x, y) \approx \delta x^2 |f|^2 \left| \frac{\partial \phi}{\partial x} \right|^2 \otimes p(x, y)$$



**DIC alignment**

<http://www.molecularexpressions.com/primer/techniques/dic/dicconfiguration.html>



**Example of DIC**

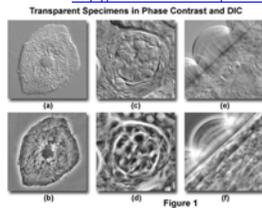


An amoeba with the numerous crystals that are found in this species. We will use this example throughout this lecture to compare methods

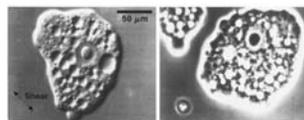
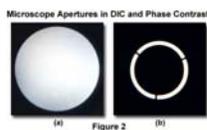
<http://www.microscopy-uk.org.uk/index.html> <http://www.microscopy-uk.org.uk/intro/index.html>

**Comparison of DIC and Phase contrast**

<http://www.molecularexpressions.com/primer/techniques/index.html>



<http://www.molecularexpressions.com/primer/techniques/dic/dicphasecomparison.html>



Fundamentals of Light Microscopy and Electronic Imaging  
Douglas S. Murphy

**Polarized Light Microscopy**

### Polarized Light Microscopy

<http://www.molecularexpressions.com/primer/techniques/polarized/polarizedhome.html>

**Figure 2**

**Figure 7**

**Example**

<http://www.molecularexpressions.com/primer/virtual/polarizing/index.html>

## Köhler Illumination For Fluorescence Microscopy Or Reflected Light

Dichroic

A ray diagram of Köhler illumination with an incident light system. L.C. = lamp collector lens; A.I. = aperture-iris diaphragm; F.I. = field-iris diaphragm; L2, L3 = relay lenses in the illuminating system; OB = objective; B.f.p. = back focal plane of the objective; P.I.P. = primary image plane; R.D. = exit pupil of the eyepiece or the 'Ramsden disc'. Note: the angles of the rays are much exaggerated for clarity; the crossing of four rays at a single point on the mirror is coincidental, and arranged thus for ease in drawing.

An Introduction to the optical microscope  
Savile Brodbery

#### Conjugate Planes in the Incident Light Optical System

Figure 6

<http://www.microscopyu.com/articles/formulas/formulasconjugate.html>

## Confocal Microscopy

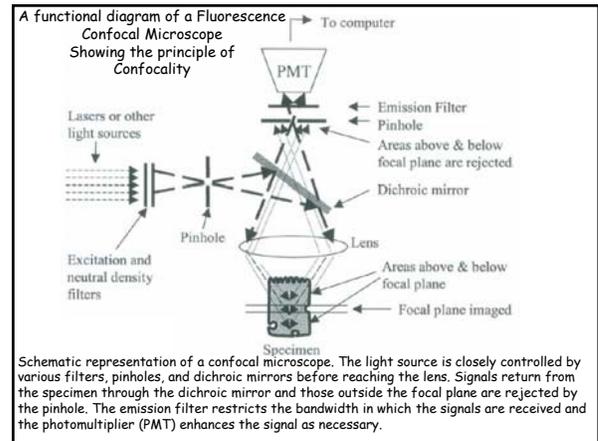
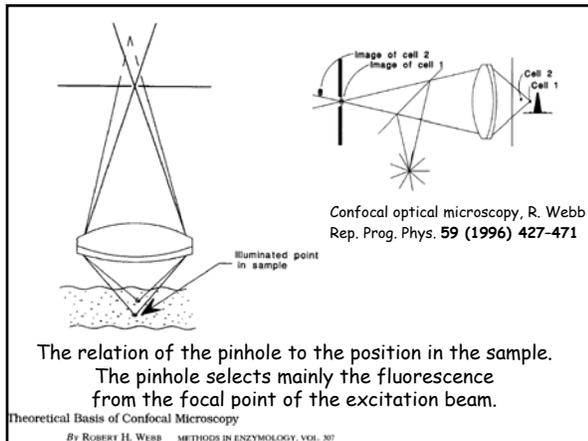
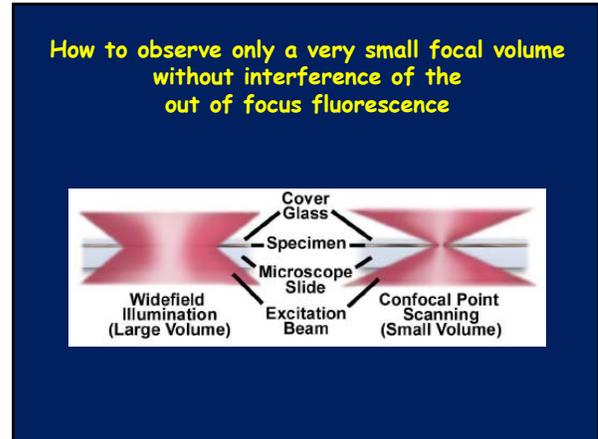
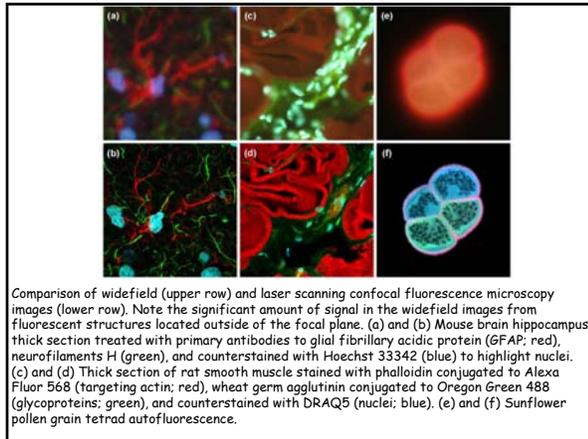
<http://www.microscopyu.com/tutorials/java/virtual/confocal/index.html>

<http://www.microscopyu.com/tutorials/flash/resonantscanning/confocalresonantscanning/index.html>

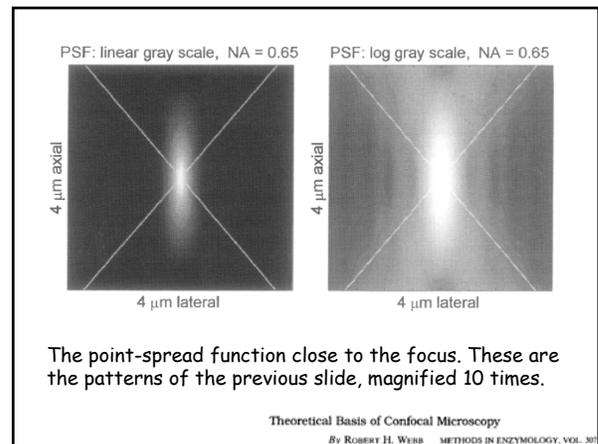
<http://www.nature.com/milestones/milelight/index.html>

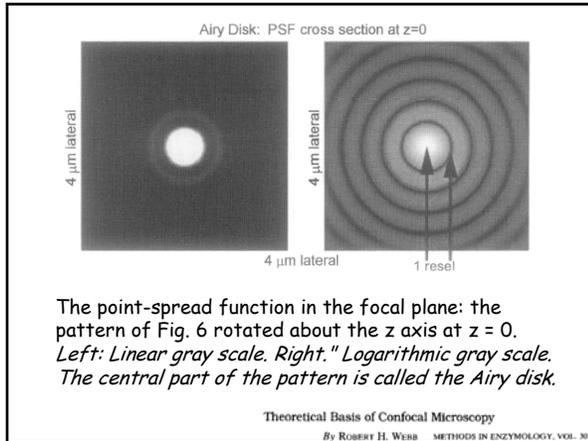
See (for many references):  
 Methods in Enzymology  
 Volume 307, Pages 3-663 (1999)  
 Confocal Microscopy  
 Edited by: P. Michael Conn

Confocal optical microscopy Robert H Webb  
 Rep. Prog. Phys. 59 (1996) 427-471



# Point Spread Functions





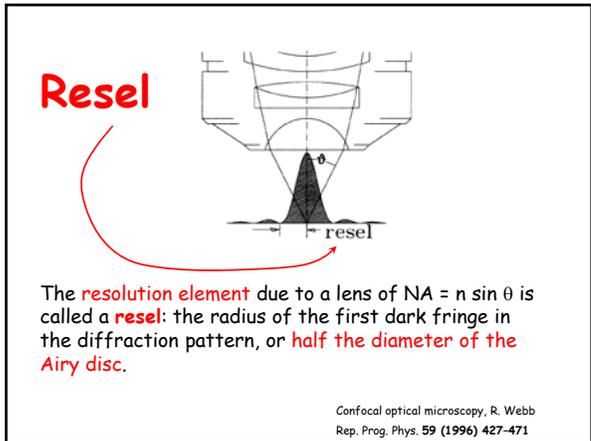
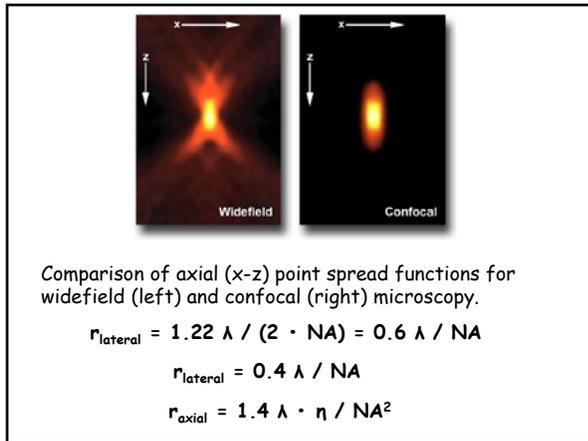
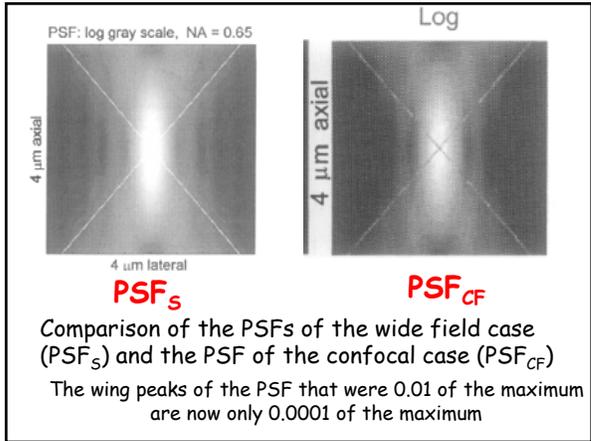
**Point spread function of illumination (from a point source).**

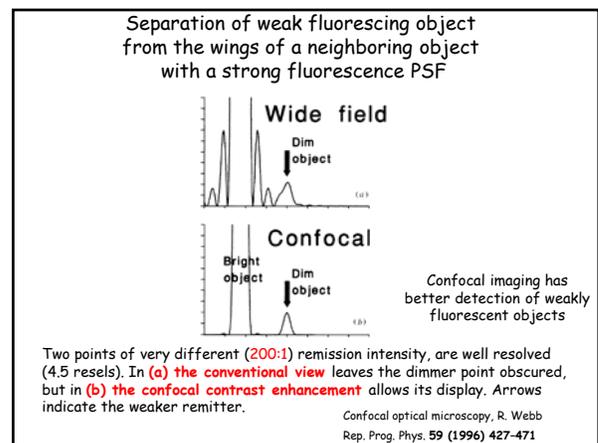
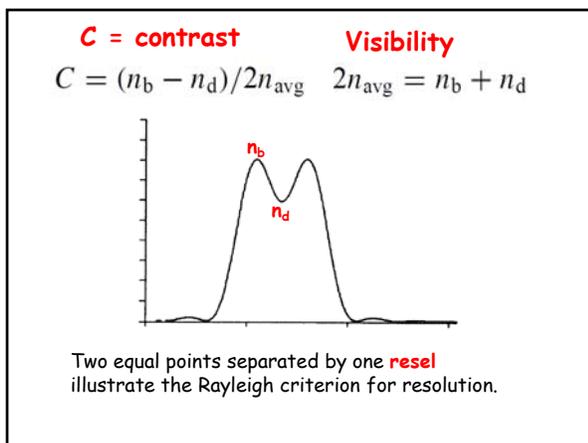
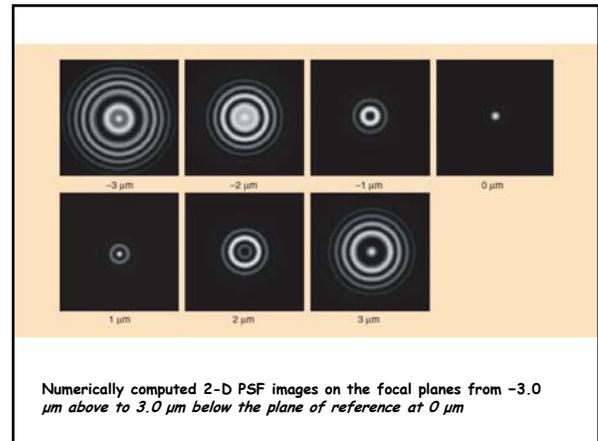
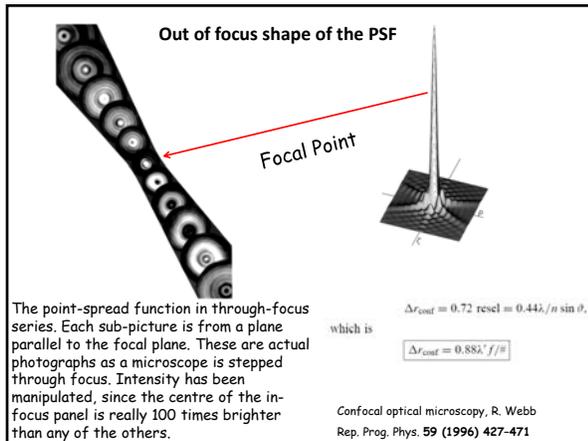
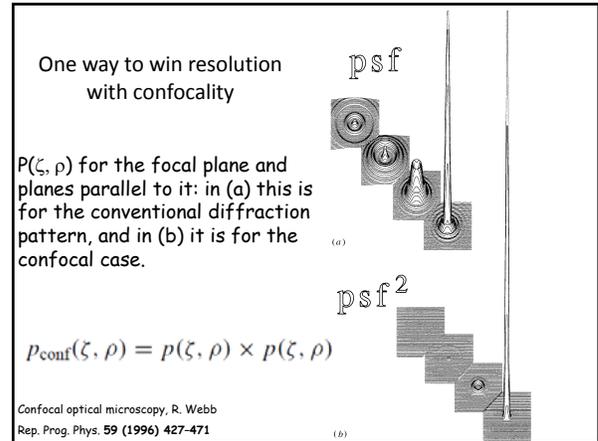
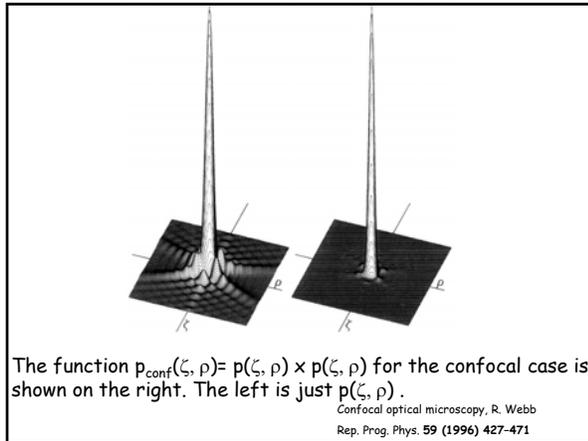
Important: The **point spread functions** discussed above are due to the **objective lens**. The complex geometric form of the point spread function (PSF) is due to **diffraction** from the **circular lens (diffraction from a circular aperture)**, using a **point source of light**. This is because of the wave nature of light. The smaller the lens aperture, the wider the pattern, and the lower the resolution. A lens with a large aperture has a much greater resolution, simply because the diffraction pattern is smaller (remember the experiment in the optical lab). The **central part of the diffraction pattern** is called the **Airy Disk**, and the size of the Airy disk sets the **spatial resolution**. The radius of this disc sets a measure of the spatial resolution.

The **confocal point spread function (PSF<sub>CF</sub>)** is a combination of the **PSF<sub>S</sub> of the illumination** (light source) and the **PSF<sub>P</sub> of the pinhole**.

$$PSF_{CF} = PSF_S \times PSF_P$$

The objective lens images every point of the sample fluorescence as a PSF at the image plane. That is, the PSF from every point of the sample is summed up at the image plane. This ends up as a **convolution of all the point spread functions from the different points at one point of the image** (see later discussion of deconvolution). But we can make the same calculation by imagining the PSF<sub>P</sub> of the pinhole imaged on the sample. This is because we are only interested in the light passing through the pinhole. These two PSF calculations, which are the distribution of the intensity of light - passing from the sample through the pinhole, or through the pinhole to the sample - are identical.

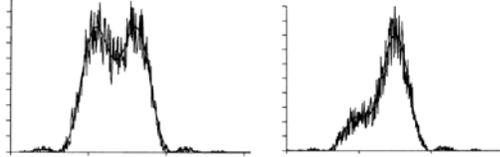




**But remember there is still "photon" noise in the acquisition**

It is not really contrast, but *noise that determines visibility*; this is due to the random arrival times of the photons—the so-called *photon noise* or *quantum noise*

$$\text{noise} = \sqrt{\text{number of photons}}$$

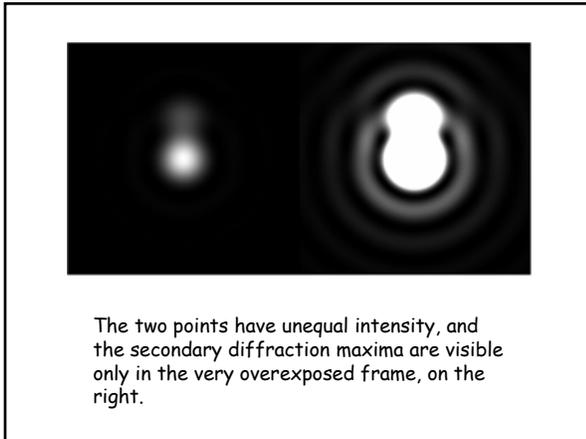


Two noisy equal points are still resolved according to the Rayleigh criterion, but we may not be able to see the dip.

Two points separated by one resel, but of different brightness and obscured by noise. These may not be resolvable at this one resel separation.

$$\text{SNR} = (n_b - n_d) / \sqrt{n_{\text{avg}}} = C \sqrt{n_{\text{avg}}} \quad N \propto \sqrt{n_{\text{avg}}} \quad C = (n_b - n_d) / n_{\text{avg}}$$

But very often it is the background noise (not the sample directly) that causes most of the noise



The two points have unequal intensity, and the secondary diffraction maxima are visible only in the very overexposed frame, on the right.

Integration of the point spread function in a full field image.  
Integrating when focusing at different planes.

$$\int_{\text{Plane}} \text{PSF} = \int_{\text{Plane}} \text{light} = \text{constant}$$

Integration of the point spread function of a confocal image

$$\int_{\text{Focal plane}} \text{PSF}_{\text{CF}} \gg \int_{\text{Any other plane}} \text{PSF}_{\text{CF}}$$

and

$$\int_{\text{Focal plane}} \text{PSF}_{\text{CF}} \gg \iint_{\text{All other planes}} \text{PSF}_{\text{CF}}$$

For the confocal point-spread function the integral is not of "light," it is of "light that reached a sample point and got back through the pinhole."

**Magnification and the size of the pinhole:**

Every lens has two point spread functions: one on each side. If the lens magnifies by a factor of 60, then the point-spread function on one side is small and that on the other side is roughly 60 times as big (roughly, because the diffraction peak details are not exactly images of each other and magnification is a concept from geometric optics). A 50x objective lens with NA = 0.85 forms a resel at the sample that is 0.4 μ across, but at the image plane of the microscope, the NA is 0.014 and the resel is 24 μ. **So there is no need to make submicron pinholes; use one that is comfortably in the 50- to 100-μ range.**

Note: If you use an new objective with low magnification but high NA, it is important to adjust pinhole size to suit the objective lens.

2-photon excitation scanning microscopy

$P_{ave}$  is the average power of the illumination beam,  $\delta_2$  is the two-photon cross section of the fluorescent molecule, and  $\lambda$  is the excitation wavelength. **1 GM (Göppert-Mayer) =  $10^{-50}$  [m<sup>4</sup> s] per photon.**

For a  $\delta_2$  of approximately 10 GM, focusing through an objective of NA > 1, an average incident laser power of  $\approx 1-50$  mW, operating at a wavelength ranging from 680 to 1100 nm with 80-150 fs pulse width and 80-100 MHz repetition rate, one should get fluorescence without saturation.

$n_{2\gamma}$  is the probability that a certain fluorophore simultaneously absorbs two photons during a single pulse; in the paraxial approximation this is given by

$$n_{2\gamma} \propto \frac{\delta_2 \cdot P_{ave}^2}{\tau_p f_p^2} \cdot \left( \frac{NA^2}{2hc\lambda} \right)^2$$

Pulse duration  $\tau_p$  and  $f_p$  repetition rate

BioMedical Engineering OnLine 2006, 5:36  
Diaspro et al; June 2006

A generic confocal microscope.

The similarities of the one-photon confocal and two-photon microscopes

A multiphoton microscope. The laser delivers all its energy in a very short pulse, so two photons can reach the sample nearly simultaneously. No pinhole is needed.

Two photon excitation: Separation of Excitation and Emission

Stokes shift  $\approx 30$  nm

dichroic mirror to block 1-photon excitation

Near infra-red wavelength of short 150 femtosecond excitation pulse

1-photon Excitation

Emission

2-photon Excitation

emission filter used with 2-photon excitation

No photolysis Outside focus

Two-photon excitation

1 photon Raman

Deep penetration into tissue; little scattering

Confocal-type localization

No Raman Interference With 2-photon excitation

With pinhole

Without pinhole

BioMedical Engineering OnLine 2006, 5:36  
Diaspro et al; June 2006

Point Spread Function comparisons

Wide field 2-photon non-confocal 1-photon confocal

Optical conditions: excitation wavelengths are 488 nm and 900 nm for 1PE and 2PE, respectively; emission wavelength is 520 nm; numerical aperture is 1.3 for an oil immersion objective with oil refractive index value set at 1.515.

BioMedical Engineering OnLine 2006, 5:36  
Diaspro et al; June 2006

Deconvolution Microscopy

**Convolution of the object with the PSF**

The general relationship between input and output

$$i(x, y, z) = N(\iiint o(x', y', z') h(x, y, z, x', y', z') dx' dy' dz')$$

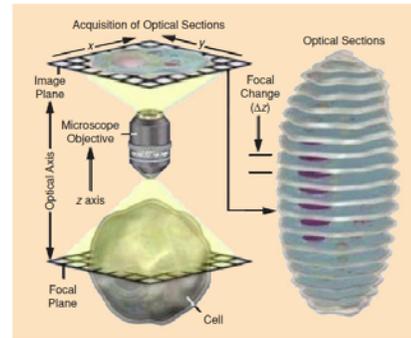
*i* is the acquired image, *o* the object acquired, *h* the PSF and *N* the noise.

Assuming linearity and shift invariance in the image formation process

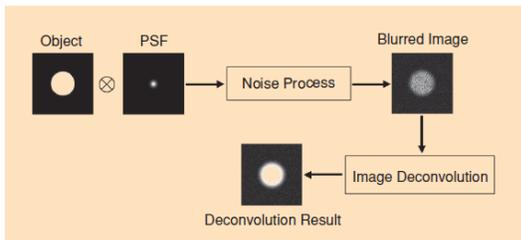
$$i(x, y, z) = N(\iiint o(x', y', z') h(x - x', y - y', z - z') dx' dy' dz')$$

This is a standard 3D convolution integral

$$i(x, y, z) = N(h(x, y, z) \otimes o(x, y, z))$$



An example of the acquired 3-D image of a cell, captured by a fluorescence microscope.



Schematic of a general deconvolution procedure

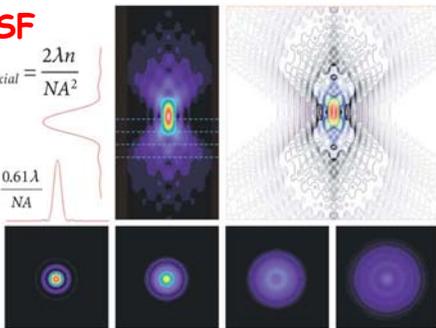
In order to correct for artifacts, you have to be able to measure or calculate or measure the correct PSF

**Ideal PSF**

$$r_{axial} = \frac{2\lambda n}{NA^2}$$

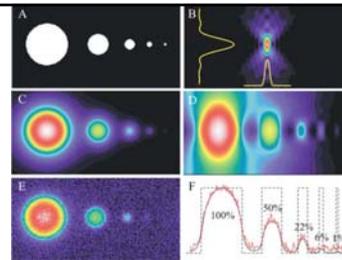
Rayleigh criterion:

$$r_{lateral} = \frac{0.61\lambda}{n \sin \alpha} = \frac{0.61\lambda}{NA}$$



Diffraction pattern of an ideal, aberration-free objective, in one, two and three dimensions

**Blur from PSF and Noise**



Three-dimensional simulation of an object observed through a microscope, in the presence of optical blur and noise. The object consists of five spheres with different diameters but the same fluorescence density: A initial object; B point spread function (PSF) of the microscope; C, D lateral and axial cross sections of the object after convolution with the microscope's PSF; E lateral cross section of the object after blurring and the addition of noise; F intensity profiles and percentage of the object's maximum intensity of original (dashed), blurred (black) and blurred-noisy (red) data. Following blurring, the smaller the object, the weaker its maximum intensity is likely to be. Noise reduces the likelihood of detecting small and highly attenuated objects

**A** Axial distance mismatch due to the presence of layers with different refractive indices along the optical path. **B** Lateral section of the bead. **C, D** Observation of a 6 μm fluorescent calibration bead without (C) and with (D) distance calibration

**Examples of artifacts: optical problem (top), oil mixing (middle) and vibrations (bottom)**

The signal-to-noise ratio can be increased significantly by applying circular averaging with respect to the optical axis

**Experimental PSF extraction:** raw data (top), and data after bead averaging and circular symmetry processing (bottom)

PSF is a function of emission wavelength, and the objective must be characterized with different color beads

PSF for Dapi, GFP and rhodamine wavelengths, using an oil with a refractive index of 1.518. Bottom: PSF for the rhodamine wavelength, using oils with refractive indices of 1.514, 1.516, 1.518 and 1.520

Ideally, PSF should be measured under the same conditions as the specimen itself.

Distortion of a PSF due to the natural fluctuation of local refractive indices. Fluorescent beads were injected into a cell before fixation. **Top:** Experimental PSF determined in the absence of a cell. **Bottom:** Distorted PSF determined from a bead within a cell. **Left:** DIC image of the cell, showing local refractive index variations; the black arrow indicates the location of the bead

**Convolution & Deconvolution**

The general relationship between input and output

$$i(x, y, z) = N(\iiint o(x', y', z') h(x, y, z, x', y', z') dx' dy' dz')$$

*i* is the acquired image, *o* the object acquired, *h* the PSF and *N* the noise.

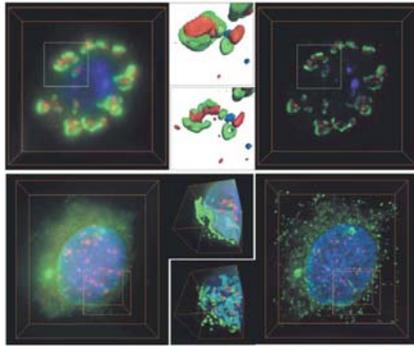
$$i(x, y, z) = N(\iiint o(x', y', z') h(x - x', y - y', z - z') dx' dy' dz')$$

$$i(x, y, z) = N(h(x, y, z) \otimes o(x, y, z))$$

The Fourier transformation of the PSF, *H*, is called the optical transfer function (OTF).

$$FFF\{h(x, y, z) \otimes o(x, y, z)\} = H(u, v, w) O(u, v, w)$$

## Deconvolving full field images with software



Before

After

(TABLE 2) GENERAL CHARACTERISTICS OF DECONVOLUTION METHODS OF 3-D FLUORESCENCE MICROSCOPY IMAGES.

TYPE	METHOD	REFERENCES	NOISE MODEL	COMMENTS
1) NO-NEIGHBORS		[6], [15]	NOT ASSUMED	SIMPLE AND FAST, REDUCE SNR AND INTRODUCE STRUCTURAL ARTIFACTS IN THE RESTORED IMAGE.
2) NEIGHBORING	NEAREST-NEIGHBORS METHODS	[1]		
	MULTI-NEIGHBORS METHODS			
3) LINEAR	INVERSE FILTERING	[6]	NOT ASSUMED	SIMPLE AND FAST, AMPLIFY NOISE, INTRODUCE RINGING ARTIFACTS AND SUPPRESS HIGH-FREQUENCY COMPONENTS IN THE RESTORED IMAGE.
	WiENER FILTERING	[16]-[17]	ADDITIVE GAUSSIAN	
	LLS	[18]		
	TRIKONOV FILTERING	[5]		
4) NONLINEAR, ITERATIVE	JVC	[1]	NOT ASSUMED	ENFORCE NONNEGATIVITY CONSTRAINT IN AN AD-HOC MANNER, INCREASE RESOLUTION IN THE ESTIMATED IMAGE BUT NOT EFFICIENT IN REMOVING NOISE.
	RLS	[16]-[19]	NOT ASSUMED	
	ICTM	[4], [8], [20], [21]	ADDITIVE GAUSSIAN	INCREASE COMPUTATIONAL DECONVOLUTION RESULT DEPENDS ON REGULARIZATION PARAMETER AND REGULARIZATION MATRIX.
5) STATISTICAL, ITERATIVE	CARRINGTON	[4], [20], [21]		RESTORE HIGH NOISE LEVEL, EM HAS SLOW RATE OF CONVERGENCE, ML CONVERGES TO NOISE ASYMPTOTICALLY, REQUIRING REGULARIZATION AT THE COST OF INCREASED COMPUTATIONAL LOAD.
	MLEM	[11], [22]-[25]	POISSON	
	PARAMETRIC ML	[9]	ADDITIVE GAUSSIAN, POISSON	APPLICABLE WHEN BOTH THE SPECIMEN AND PSF ARE IN PARAMETRIC FORM. GAUSSIAN OR ENTROPY PRIOR DISTRIBUTION OF THE OBJECT IS ASSUMED, AND HENCE GIVES BETTER PERFORMANCE, EMPLOY REGULARIZATION AND THE CHOICE OF REGULARIZATION PARAMETER DETERMINES THE TRADE-OFF BETWEEN AMOUNT OF SMOOTHNESS AND FITTING ACCURACY.
6) BLIND DECONVOLUTION	BDRL	[7], [24], [29]-[33]	POISSON	LARGE NUMBER OF PARAMETERS ARE INVOLVED, HENCE COMPUTATIONALLY INTENSIVE.

IEEE SIGNAL PROCESSING MAGAZINE Pinaki Sarder and Arye Nehorai ,  
Deconvolution Methods for 3-D Fluorescence Microscopy Images pp. 32-45,  
MAY 2006

**“IMAGING SYSTEMS ALWAYS DISTORT OBJECTS. IMAGE DECONVOLUTION RESTORES OBJECTS TYPICALLY BY CHARACTERIZING THIS DISTORTION.**

A limitation of most algorithms is the underlying assumption of the shift-invariance property of the microscope system, which is not true in practice.”

IEEE SIGNAL PROCESSING MAGAZINE Pinaki Sarder and Arye Nehorai ,  
Deconvolution Methods for 3-D Fluorescence Microscopy Images pp. 32-45,  
MAY 2006